10-5-2020

Modelling the Effects of Complacency and Educational Countermeasures on the Spread of HIV

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Introduction

Human immunodeficiency virus (HIV) is a virus that attacks the CD4+cells in the human body, which are a critical component of the human immune system. As these CD4+cells are attacked, the immune response of the body weakens, making an individual more susceptible to disease and infection. Once the count of CD4+cells fall below 200 per cubic millimeter of blood, an individual is diagnosed with acquired immunodeficiency syndrome (AIDS) [4]. Currently, there are an estimated 1.1 million individuals living with HIV in the United States, with an average of 39,000 new cases reported every year. An estimated 14% of these individuals are unaware that they even have HIV [1]. An effective way of getting HIV infected individuals who are unaware of their serostatus on anti-retroviral therapy (ART), which lowers the viral load in the body, is through the test-and-treat method. Test-and-treat is a strategy used by governments and health organizations where populations at high-risk for being infected with HIV are targeted and screened for HIV, and any individuals who are found to have HIV are placed on ART. Test-and-treat strategies have been shown to reduce new HIV infections by up to 30% in populations where they are implemented [2]. Despite these successes, people are prone to ignoring public health threats if they do not feel personally affected by the threat; this is complacency in action. The goal of our project is to study the long-term dynamics of a system of ordinary differential equations which describes the effects of implementing test-and-treat strategies on an HIV-susceptible population.

Model Building

Our model extends the work of Okosun et al. [3] to include an educational awareness campaign as an effort to lower complacency. We model the structure of the complacency term after a Holling Type-II functional response. In the following system of equations, we describe the dynamics of the susceptible (S), infected unaware (IU), infected aware (IT), infected on ART (H), and AIDS (A) populations.

$$\frac{dS}{dt} = \lambda - \frac{\left(1-\frac{\eta A}{k+A}\right)(\beta_1 I_U + \beta_2 I_T + \beta_3 H)S}{N} - \mu$$

$$\frac{dI_U}{dt} = \frac{\left(1-\frac{\eta A}{k+A}\right)(\beta_1 I_U + \beta_2 I_T + \beta_3 H)S}{N} - (\theta+\delta+\mu)I_U$$

$$\frac{dI_T}{dt} = \theta I_U - (\delta+\mu+\pi)I_T$$

$$\frac{dH}{dt} = \pi I_T - (\sigma+\delta+\mu)H$$

$$\frac{dA}{dt} = \delta I_U + \delta I_T + \sigma\delta H - (\alpha+\mu)A,$$

where $N = S + I_U + I_T + H + A$.

Model Analysis

The first step in analyzing our model was to find the disease-free equilibrium by setting our system of differential equations equal to zero:

$$E_0 = (\frac{\lambda}{\mu}, 0,0,0,0)$$

Then, we found the reproductive ratio, R_0, using the next-generation matrix method.

$$F = \begin{bmatrix} 0 & -\beta_1 & -\beta_2 & -\beta_3 & 0 \\ 0 & \beta_1 & \beta_2 & \beta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} \mu & 0 & 0 & 0 & 0 \\ 0 & \delta+\theta+\mu & 0 & 0 & 0 \\ 0 & -\theta & \delta+\mu+\pi & 0 & 0 \\ 0 & 0 & -\pi & \mu+\delta\sigma & 0 \\ 0 & -\delta & -\delta & \delta\sigma & \alpha+\mu \end{bmatrix}$$

$$G = FV^{-1}$$

$$R_0 = \rho(G) = \frac{\beta_3\theta\pi + \beta_2\theta(\mu+\delta\sigma) + \beta_1(\delta+\mu+\pi)(\mu+\delta\sigma)}{(\delta+\sigma+\mu)(\delta+\mu+\pi)(\mu+\delta\sigma)}$$

Next, we proved global stability for E_0 when $R_0 < 1$.
Proposed Lyapunov function:

$$L = I_U + \left(\frac{\beta_2}{\delta+\mu+\pi} + \frac{\beta_3\pi}{(\delta+\mu+\pi)+(\mu+\sigma\delta)}\right)I_T + \frac{\beta_3}{\mu+\sigma\delta}H$$

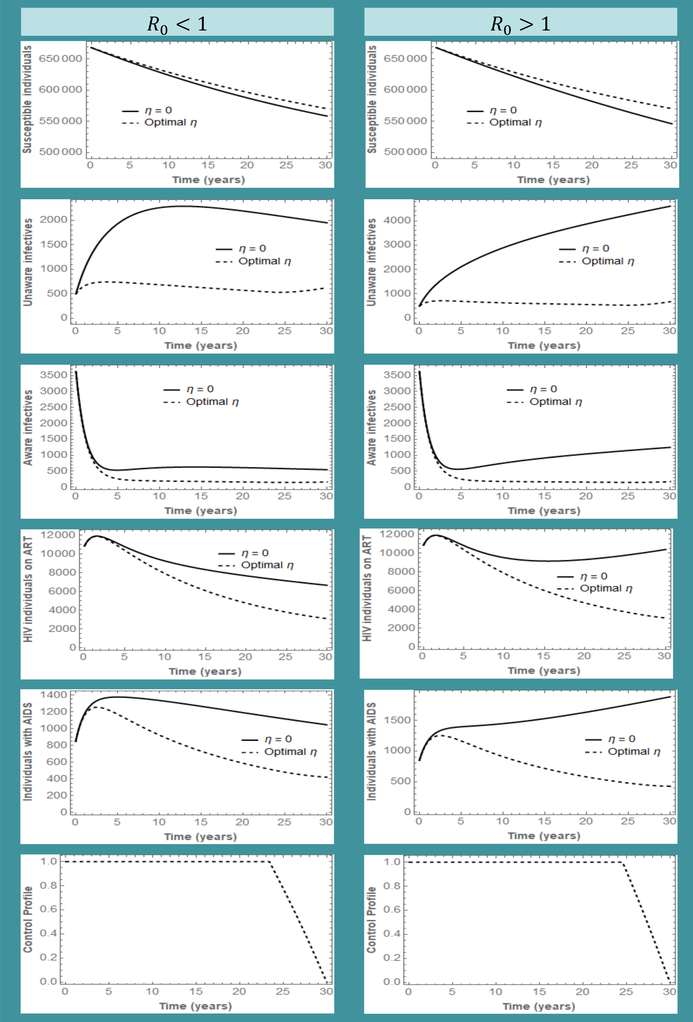
$$\frac{dL}{dt} = (\theta+\delta+\mu)(R_0-1)I_U$$

Since $\frac{dL}{dt}$ is negative when $R_0 < 1$, we can conclude that I_U, I_T, and H approach 0, which causes A to approach 0 and S to approach $\frac{\lambda}{\mu}$, thus proving global asymptotic stability when $R_0 < 1$.

Next, we allowed the complacency parameter to be time-dependent and formed a related optimal control problem for our model. Our goal was to find an optimal plan of educational campaigning that would minimize the number of people infected with HIV, which would also minimize the cost of implementing such a campaign. To do so, we replaced the complacency variable *η,* which tracked the pro-activeness of the population to avoid encounters where they could potentially acquire HIV, with the function *u(t),* which represented the educational campaign. We then used Pontryagin's Maximum Principle to characterize optimal control profiles. We numerically simulated the results of optimal control using a fourth-order Runge-Kutta forward-backward sweep method. These results can be seen in the graphs on the right.

Discussion

Illustrated by numerical simulations that follow, when $R_0 < 1$ and the optimal educational campaign is applied, the number of people living with HIV, but who are unaware of it, is a fourth of what it would have been without education. When $R_0 > 1$ and the optimal educational treatment is applied, after 30 years, the number of people living with HIV, but are unaware of it, is similarly reduced. This demonstrates how impactful an educational campaign could be in decreasing the number of HIV cases in a population.



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Acknowledgements

This project is supported by an Institutional Development Award (IDeA) from the National Institute of General Medical Sciences (2 P20 GM103499 15) from the National Institutes of Health. Also, we would like to thank the South Carolina INBRE Program, Winthrop University Summer Undergraduate Research Experience Program (SURE), and the Winthrop Department of Mathematics for both the support and funding of this project.